Multi-Axial Sliced Finite Element Model for Toroidal Inductors

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This paper demonstrates a novel approach for analyzing 3-D electromagnetic fields in toroidal inductors with minimal computational time and resources. A 2-D magnetodynamic finite element (FE) problem is solved in several axial and radial slices of the 3-D inductor geometry using the AVI-formulation. A procedure to couple the slices with each other through circuit equations and suitable interface conditions is proposed. The obtained results are validated with a 3-D FE model in a time-harmonic case. The modelling shows massive reduction in computation time compared to traditional 3-D FE analysis.

Index Terms— Eddy currents, finite element analysis, inductor, proximity effect, skin effect, winding loss, winding resistance.

I. INTRODUCTION

THE IMPROVEMENTS in semiconductors and soft-switching topologies have made higher switching frequency operations possible for power converters. Such high frequency operations drastically influence the design of magnetic components in power electronics. Especially, the presence of skin and proximity effects in the windings of magnetic components cannot be neglected at high switching frequencies.

The skin effect is a result of eddy currents flowing under the influence of the local magnetic field of a conductor, while proximity effect refers to eddy currents induced by an external magnetic field. In particular, under presence of such frequency dependent effects, windings of power magnetic components share the highest percentage of loss at high frequencies [1]-[3]. For getting most optimal designs, skin and proximity effects need thorough treatment during modelling stage.

At high frequencies, eddy current effects in windings with multiple strands (sub-conductors) become too complex for conventional analytical methods [4], [5]. Parameters like porosity factor and changing penetration depth require tuning of analytical methods based on experiments or heavy numerical simulations [6].

Several loss models have been developed to reduce the computation times and to reach accuracy levels as high as possible [7]-[10]. However, precise analytical models of the fields in problems like windings with multiple stranded conductors or any asymmetrical winding structures are missing [11]. This makes it difficult for industry to completely rely on analytical or empirical methods for design of magnetic components. Hence, numerical methods based on two dimensional (2-D) and three dimensional (3-D) approaches are the preferred alternatives for better reliability and accuracy. Although 2-D finite element (FE) method (FEM) is an ideal choice considering computational time and cost, the traditional 2-D FEM stays limited to simple structures. Therefore, to attain high level of precision, the 3-D FEM has to be used for optimization.

3-D FE analysis of eddy currents and circulating currents in the inductors and transformers with multiple parallel conductors would require fine 3-D discretization of each wire. The entire problem becomes substantially big for solving in a single computer system. For addressing these issues, there have been concrete efforts in [12] and [13] for analyzing complex winding structures. An approach combining rotationally symmetric 2-D and 3-D simulation was presented in [12]. The method presented in [13] is based on the partial element equivalent circuit (PEEC). For the precise computations in case of inductors with ferromagnetic cores, PEEC has been combined with the boundary element method in [14] and with FEM in [15]. The method is consistently being developed for accurate modelling of linear inhomogeneous conductive and magnetic media [16]-[19]. PEEC is also extended for different loss models for coreless inductors [20]. However, for exploring inhomogeneous distribution of current density in winding conductors at high frequency, FEM is still seen as more robust and reliable approach [5], [21].

This paper presents a computationally efficient and optimal method for exploring 3-D eddy current effects in the winding of toroidal inductors used in power conversion units. The idea is based on coupled solution of electromagnetic fields in 2-D slices taken axially and radially from the inductor. The approach will be called a multi-axial slice model (MASM).

As a test case, a single symmetry sector of a toroidal inductor with an equally distributed winding is considered. Since the paper targets on providing the concept of MASM, two simplified windings with one and three parallel sub-conductors are considered. The MASM is created in the MATLAB environment. For validating the proposed modelling approach, comparison is made against 3-D simulations from COMSOL Multiphysics. Section II covers the theoretical aspects about the MASM. It also includes details about the geometry and supporting technical aspects for the simulations. Along with the necessary results, Section III provides in-depth comparative analysis followed by conclusions in Section IV. We widely use the definitions explained in [22], [23].

II. MODELLING METHODOLOGY

A. Geometric model

Fig. 1 shows the considered toroidal inductor as well as one of its symmetry sectors. Both the MASM and 3-D FEM are built for the symmetry sector. A linear ferrite core with relative
permeability $\mu_r = 3000$ is considered. The inner and outer radii of the core are designated by $r_{in}$ and $r_{out}$, respectively.

The height of the core is $h$. A copper conductor with 0.6 mm diameter is used. The inductor carries $N = 50$ turns equally distributed over the periphery as shown in Fig.1. The symmetry sector covers an angle of $\phi_{sym} = 2\pi/N$.

In Fig. 1, the symmetry sector is sliced with four planes, two axial ones in green ($z$ = constant) and two radial ones in blue ($r$ = constant), so that the current-carrying conductors are approximately perpendicular to each slice. The axial slices allow accounting for the stray field on the top and bottom of the core. The core is only considered in the axial slices, not in the radial ones.

In general, we can consider $n_{axi}$ axial slices and $n_{rad}$ radial slices, indexing the slices with $k = 1, \ldots, n_{axi}+n_{rad}$. We assume that the winding consists of $n_{par}$ parallel conductors, which means that each 2-D slice will include $2n_{par}$ distinct conductor regions corresponding to the positive and negative coils sides. The conductor regions $\Omega_{kq}$ are indexed with $q = 1, \ldots, 2n_{par}$ and the parallel paths with $p = 1, \ldots, n_{par}$.

B. Equations in one slice

The magnetic field in each slice $k$ is mostly parallel to the slice plane, and can be analyzed comfortably using 2-D FE analysis. In this paper, the AVI formulation [24] is used for the field analysis. The governing equations for the 2-D electromagnetic field in slice $k$ associated with a perpendicular length $l_k$ are given by

$$- \nabla \cdot \nabla A_k = 0$$

outside the conductors, and

$$- \nabla \cdot \nabla A_k + \sigma \frac{\partial A_k}{\partial t} - \sigma \frac{u_{kn}}{l_k} = 0$$

in $\Omega_k$. In the equations, $A_k$ is the component of the magnetic vector potential perpendicular to the slice, $u_{kn}$ is the potential difference in $\Omega_k$, $\nu$ is the reluctivity and $\sigma$ is the electrical conductivity. The current in the conductor branch $p$ is given by

$$i_p = - m_{kqp} \int_{\Omega_k} \sigma \frac{\partial A_k}{\partial t} d\Omega_k + m_{kqp} \frac{u_{kn}}{l_k}$$

for each $k$ (3)

where $m_{kqp}$ associates the currents $p$ with regions $\Omega_{kq}$ according to

$$m_{kqp} \begin{cases} +1, & \text{if current of branch } p \text{ flows in the positive direction through } \Omega_{kq} \\ -1, & \text{if current of branch } p \text{ flows in the negative direction through } \Omega_{kq} \\ 0, & \text{otherwise} \end{cases}$$

and $R_{kq}$ is the resistance of corresponding to domain $\Omega_{kq}$. It is emphasized that the currents are common for each slice, so that $i_p$ is independent of $k$. Now based on (3), the voltage over conductor domains $\Omega_{kq}$ is expressed as

$$u_{kq} = m_{kqp} R_{kq} i_p + R_{kq} \int_{\Omega_k} \sigma \frac{\partial A_k}{\partial t} d\Omega_k$$

(4)

The voltage over each conductor $p$ in each slice $k$ is

$$U_{kp} = \sum_{q} m_{kqp} u_{kq}$$

(5)

Discretizing (2) and (4) with the Galerkin method yields

$$\begin{bmatrix} S_k + T_k \frac{d}{dt} D_{kq} & 0 \\ C_{kq} \frac{d}{dt} - I R M_k & 0 \end{bmatrix} \begin{bmatrix} a_k \\ i \end{bmatrix} = \begin{bmatrix} 0 \\ U_k \end{bmatrix}$$

(6)

where $a_k$ contains the nodal values of $A_k$, $u_k$ contains $u_{kn}$ and $i$ contains $i_p$. $S_k$ and $T_k$ are the stiffness and damping matrices respectively, $D_{kq}$ and $C_{kq}$ are related to the field source and the back-emf induced to the conductors, respectively. They are obtained as

$$[S_k]_{kq} = \sum_{w} w_k d\Omega_{wq}$$

(7)

$$[C_{kq}]_{kq} = \sigma R_{kq} \int_{\Omega_k} w_k d\Omega_{kq}$$

(8)

where $w_k$ is the FE shape function associated with node $n$. $M_k$ represents the coupling matrix for conductor domains and parallel branches according to (5), and $R_k$ is the diagonal matrix for conductor resistances $R_{kq}$.

$$[M_k]_{kq} = m_{kqp}$$

(9)

$$[R_k]_{kq} = R_{kq}$$

(10)

C. Multi-axial slice model and simulation

As shown in Fig.2, the toroidal inductor is assumed to be placed in the cylindrical coordinate system $r$-$\phi$-$z$ so that the core covers the region $[-h/2, h/2] \times [r_{max}, r_{min}] \times [-\phi_{sym}/2, \phi_{sym}/2]$. The axial slices are equally-sized circular sectors $[0, r_{max}] \times [-\phi_{sym}/2, \phi_{sym}/2]$ chosen from axial positions $z_k$ corresponding to $n_{arc}$ Gauss quadrature points over $z \in [-h/2, h/2]$. The relative perpendicular lengths $l_k / h$ correspond to Gauss integration weights. The fields in the axial slices are described in $x$-$y$
Axial and radial slices. The coupling includes three conditions:

1. Forcing the total currents in each conductor to be equal in each slice.
2. Forcing the tangential magnetic fields $H \cdot u_p$ at the interfaces between the radial and axial slices to be continuous in the weak sense.
3. Accounting for the perpendicular flux crossing the interfaces between the radial and axial slices.

Condition 1 is satisfied automatically in the AVI formulation, since the currents are common for each slice. Condition 2 is implemented through a non-homogeneous Neumann condition in the radial slices $k$ as

$$\int_{\Gamma_{rad}} w_x H \cdot u_p d\Gamma = \int_{\Gamma_{rad}} w_x H \cdot u_p d\Gamma$$

where $\Gamma_{rad}$ is the boundary of radial slice $k = n_{axi} + 1$, ..., $n_{axi} + n_{rad}$ at the top ($z = h/2$) or bottom ($z = -h/2$) surface of the core, and $\Gamma_{axi}$ is the corresponding boundary in the top or bottom axial slice. However, to avoid complex interpolations between two possibly non-conforming meshes, we derive here a simple approach by approximating the circumferential field strength in radial slice $k$ as

$$H \cdot u_p = \frac{NI}{2\pi r_k}$$

where

$$I = \sum_{p=1}^{n_{par}} I_p$$

is the total current carried by the $n_{par}$ parallel conductors in one symmetry sector. When (14) and (15) are substituted in right-hand-side of (13), a non-homogeneous Neumann condition for the radial slices is obtained. The FE block matrix thus becomes

$$\begin{bmatrix}
S + T \frac{d}{dt} & D_\alpha & D_i \\
C_\alpha \frac{d}{dt} -I & RM & 0 \\
0 & M^T & 0
\end{bmatrix}
\begin{bmatrix}
a \\
u \\
i
\end{bmatrix}
= \begin{bmatrix}
0 \\
U
\end{bmatrix}$$

(16)

where the additional matrix $D_f$ is a vertical assembly of matrices

$$\begin{bmatrix}
D_{r,k} \end{bmatrix}_{axi} = \frac{N}{2\pi r_k} \int_{\Gamma_{rad}} w_x d\Gamma$$

(17)

for all $p$ and for $k = n_{axi} + 1$, ..., $n_{axi} + n_{rad}$, which account for the tangential field strength in the radial slices.

Condition 3 could perhaps be implemented by considering the flux crossing the interface as a non-zero divergence of the flux-density in the axial slices. However, this would be challenging to implement. We thus again derive a simpler approach using the Poynting theorem [25], based on which the power passing through the interface between the radial and axial slices is given by

$$P = \int_{\Gamma_{axi}} \int_{\Gamma_{rad}} r E \times H \cdot u_p d\phi dz$$

(18)

where $E$ is the electric field strength. Since the radial slices are placed at the Gauss quadrature points, we can write the integral as

$$P = \sum_{k=n_{axi}}^{n_{axi} + n_{rad}} l_k \int_{\Gamma_{rad}} E \times H \cdot u_p d\Gamma$$

(19)
Using (10) and

\[ E = -\frac{\partial A}{\partial t} \cdot u \]  

(20)

we get

\[ P = -NI \sum_{k=n_{axi}+1}^{n_{rad}} \frac{l_k}{2\pi r_k} \int \frac{\partial A}{\partial t} \, d\Gamma \]  

(21)

This power should be seen as a change \( \Delta u_p \) in the conductor potential differences, such that

\[ P = NI \Delta u_p \]  

(22)

meaning that

\[ \Delta u_p = -\sum_{k=n_{axi}+1}^{n_{rad}} \frac{l_k}{2\pi r_k} \int \frac{\partial A}{\partial t} \, d\Gamma \]  

(23)

for all \( p \). This voltage is added to the voltage equation in (5), yielding a final system of

\[
\begin{bmatrix}
S + T \frac{d}{dt} D_0 & D_x \\
C_0 \frac{d}{dt} & -I & RM & 0 \\
C_t \frac{d}{dt} & M^t & 0 & U
\end{bmatrix}
\begin{bmatrix}
a \\
u \\
i
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
U
\end{bmatrix}
\]  

(24)

where the additional matrix \( C_t \) is a horizontal assembly of matrices

\[ C_{t,k} = \frac{1}{N} D_{t,k} \]  

(25)

for all \( p \) and for \( k = n_{axi} + 1, \ldots, n_{axi} + n_{rad} \), which account for the power coming from the radial slices. Note that

\[ C_{t,k} = \frac{1}{N} D_{t,k} \]  

(26)

Equation (24) represents the whole MASM system, where the slices are coupled together.

D. 3-D simulation

Time-harmonic 3-D FE simulation with COMSOL Multiphysics is used to validate the proposed MASM. Two test cases with \( n_{rad} = 1 \) and \( n_{rad} = 3 \) parallel conductors are simulated. The respective models are represented in Fig. 3. The multiple sub-conductor case provides better insight about frequency dependent power losses in each conductors. The chosen geometries are symmetric with respect to the \( z = 0 \) plane, and thus only the lower half \( z \leq 0 \) is considered in the 3-D model for minimizing the required computation time and resources. Both halves are considered in the MASM.

A boundary layer mesh is used in the conductor section for accurately capturing the influence of skin effect on power losses. Periodic boundary conditions are used on the sides of the symmetry sector. This ensures the continuity of flux density along the circumferential path of toroid. A voltage is imposed between the inner and outer conductor cross sections in the \( z = 0 \) plane. The currents and power losses are computed in each conductor for both cases and compared to those obtained from the MASM.

III. RESULTS

The simulation results presented here are computed on a single Windows machine with 32 GB RAM and Intel Core i7-8650U (4 cores 8 threads) 1.9 GHz processor. All simulations including 3-D FEM are with linear discretization. The currents and power losses of the MASM and the 3-D FEM for the single-conductor case are shown in Fig. 4. The values are computed over a frequency range from 10 kHz to 1 MHz for sinusoidal voltage input. The current is kept constant by maintaining constant voltage to frequency ratio for all frequencies. This allows good insight on the change in losses as a function of frequency. Each graph shows data for three simulations: the reference results from 3-D FEM, results from the MASM with two axial and two radial slices, as well as results from the MASM with two axial slices, but no radial slices. The last case is done for studying the significance of including the radial slices into the model. In all simulations, the losses increase significantly above 60 kHz. As the frequency increases, the MASM without radial slices underestimates the losses. Thus, simple 2-D FEM fails to capture high frequency 3-D effects in the windings. On the other hand, the results from the MASM with both axial and radial slices agree well with those from the 3-D FEM.

Fig. 5 compares the current density distributions in the axial slices of the MASM to those obtained from the 3-D model in identical locations at 100 kHz. Similar comparison for the current densities in the radial slices is shown in Fig. 6. In this case, distribution of the current density is similar in all slices. The reason is that the conductor is perfectly aligned in the radial and the axial directions. Moreover, in a single conductor case, the skin effect plays the most influential role in the current distribution.

Similar results are produced for the winding with triple sub-conductors. The computed results for the total current and losses are shown in Fig 7. The results from the MASM are very close to the 3-D FEM computations. Again, neglecting the radial slices fails to provide precise information on frequency dependent power loss.

Along with skin effect losses, the triple conductor case also include loss components from proximity effect and circulating currents [26], [27]. The resultant distribution of the current density is shown in Figs. 8 and 9. The former depicts current...
density in axial direction while the latter shows radially directed current densities. The share of current among the parallel conductors changes as a function of the frequency. The computed currents and losses for each sub-conductor are shown in Fig. 10. The red curves indicate conductor 1, located closest to the core. The conductor 2 and 3 quantities are represented by blue and green curves respectively. As conductor 2 lies right above conductor 1, it has the largest distance from the surface of the core among the conductors. It shares the smallest amount of current at higher frequency, which explains lower AC losses than in the other two conductors. Conductor 3 is located at the intermediate distance between conductors 1 and 2 from the surface of core. For the entire frequency range, the current and power losses of conductor 3 stay between the respective values of conductors 1 and 2.

By having a closer view at the results from simulation with only 2 axial slices, but no radial ones, the computed current values deviate from the 3-D FEM quantities. For the frequency range from 10 kHz to 500 kHz, the currents have higher deviation. At lower and higher frequencies, the currents match well with the 3-D FEM. However, the scenarios are different with power losses. The computed values of losses with the MASM are quite close to the ones from 3-D FEM. The accurate understanding of such quantities is important for deciding the optimal number of parallel conductors in windings [28]. In the same context, simple 2-D FEM with only axial slices does not provide the required accuracy level at high frequencies. The simulation times for all frequencies were observed in each simulated case. On average, one MASM simulation took 4.91 seconds while one 3-D FEM simulation took 73.4 seconds in the single conductor case. In the triple conductor case, the
MASM took 4.95 seconds while the 3-D FEM took 1325 seconds. The speedup ratio of 3-D FEM and MASM computations are shown in Fig. 11. It is clearly seen that in the single conductor case, the MASM is about 15 times faster compared to 3D FEM. A more significant difference is seen in the triple conductor case, where the MASM is 265.7 times faster on an average scale compared to 3-D FEM analysis. In comparison to 3-D FEM, the computed power losses with MASM are on average less by 2.8% and 3.5% for single and triple conductor cases, respectively.

IV. CONCLUSION

The method proposed here carries potential to replace the usage of 3-D FEM approach for the toroidal inductor modelling. With a reasonable trade-off between accuracy level and simulation time, the proposed multiaxial slice model is promisingly fast. The frequency dependent power losses for multiple conductor winding can easily be analyzed without stressing available computational resources. The obtained results have shown good agreement with the results from 3-D FEM simulation of commercial software.

Based on the results, the implementation of this idea can give an extra edge to industries in cutting down massive amount of time in their design process. As a part of future work, twisting effect of multiple conductor winding for toroidal inductors will be incorporated in the developed method.

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